Method for the Fuel-Optimized Selection of a Thruster Configuration

The present invention relates to a method for the fuel-optimized selection of a configuration of thrusters on a spacecraft.

Such a method is known for example from US 6,347,262 B1 for the case of a spin-stabilized spacecraft. A configuration of thrusters on a spacecraft, as considered by the present invention, serves in particular the attitude and position correction of the spacecraft. Such an attitude and position correction via the thrusters is known for example from EP 0 750 239 A2.

From EP 0 977 687 we know of a special method for the low-fuel control of an arrangement of thrusters on a spacecraft, wherein for the purpose of finding a low-fuel solution for the control of a convex linear optimization problem is resolved through

- an initialization phase for finding a first permissible solution to the linear optimization problem and

- a subsequent iteration phase, in which, proceeding on the permissible solution to the linear optimization problem, an iterative optimization of an effectiveness criterion takes place.

In this method a dual simplex algorithm is applied, which is supposed to find an optimal solution to the problem through a largely unfocused search method, wherein however it is possible with this method that there is no solution for the currently existing force-momental vector and the present thruster arrangement.

From N. Karmakar: A new polynominal time algorithm for linear programming, Combinatorica 4 (4), 1984, p. 373-395 we know of a basic method for solving linear optimization problems of a general form.

It is the object of the present invention to offer a method for the fuel-optimized selection of a configuration of thrusters on a spacecraft, which permits a focused search of a solution to the linear optimization problem that is permissible in any case. This object is achieved through the features of patent claim 1.

The present invention relates to a method for the fuel-optimized selection of a configuration of thrusters on a spacecraft, wherein for the purpose of finding a fuel-optimized solution for the selection process a linear optimization problem, particularly a convex linear optimization problem, is resolved, through

- an initialization phase for finding a first permissible solution to the linear optimization problem and

- a subsequent iteration phase, in which proceeding on the permissible solution to the linear optimization problem an iterative optimization of an effectiveness criterion occurs.

Pursuant to the invention it is provided that

- in each iteration step a scaled iteration gradient is formed and

- the iteration gradient is multiplied with a limiting factor for a maximum iteration threshold, which is formed while taking at least one boundary value condition for a permissible solution into account.

By applying a scaled iteration gradient instead of a mere search method as in the state of the art, a focused locating process for the optimal solution can take place. When forming the scaled iteration gradient, again especially at least one boundary value condition for a permissible solution can be included. Application of a scaled iteration gradient also largely excludes the circumstance that only a suboptimal solution to the linear optimization problem is found. The fact that linear problems can involve particularly a so-called convex problem is basically known, see for example the chapter

"Linear Programming" at the following internet link of the European Business School of the Schloss Reichartshausen University:

http://www.ebs.de/Lehrstuehle/Wirtschaftsinformatik/NEW/Courses/Semester2/Math2/.

By taking the boundary value conditions into account within the framework of the limiting factor it is achieved that as the respectively next iteration solution again a solution is determined, which is within a permissible range of values, because the limiting factor allows the iteration step width to be adjusted accordingly so that a boundary value condition is not violated. With a possible consideration of the boundary value conditions within the framework of forming the scaled iteration gradient, in particular the gradient direction can be selected substantially such that once again a solution that is within a permissible range of values is determined as the respectively next iteration solution.

Furthermore, for the present linear optimization problems it is known that the optimal permissible solution, which corresponds to a coordinate point in a multidimensional space of all permissible solutions that is limited by boundary conditions, is located on the boundary of said limited space. In this manner the scaled iteration gradient and the limiting factor are preferably adjusted such that an iterative approximation of an optimal point on the delimitation of the multidimensional space of the permissible solutions occurs.

It can now be provided in particular that an upper bound for a permissible solution is defined as a boundary value condition.

Moreover it can be advantageously provided that the iteration gradient is determined with the help of a Gauss elimination, which represents a very fast method.

It can also be provided in particular that in every iteration step a scaling of the iteration gradient takes place such that a gradient component becomes smaller the closer the corresponding component of the result of the previous iteration step comes to a boundary value condition. In this way a new scaling operation of the iteration gradient is performed in every iteration step, wherein certain components of the gradient disappear

when the corresponding components of the previous iteration solution come very close to a boundary value condition, i.e. for example they become smaller than a first predefined distance. Said first distance can also be selected to be infinitesimally small.

Furthermore it can be provided that the iteration phase is terminated as soon as the result of an iteration step exceeds at least one boundary value condition and that the result of the previous iteration step is determined as an optimal solution of the effectiveness criterion. Thus the iteration is terminated if the algorithm leaves the range of permissible solutions, and the last permissible solution is determined as the optimal solution. In this way it is guaranteed in a simple manner that in any case the solution that is determined as the final result of the method is as optimal as possible and is simultaneously permissible.

The iteration phase, however, can also be terminated as soon as the iteration method converges against a permissible solution and the result of a certain iteration step differs from the result of a previous iteration step by less than a second predefined distance, wherein the result of the last iteration step is determined as an optimal solution of the effectiveness criterion.

The following explains one specific embodiment of the present invention.

A method for the fuel-optimized selection of an arrangement of thrusters on a spacecraft is considered, which is used for attitude and position control of the spacecraft. In order to generate forces and moments that are applied on a spacecraft, for example in order to be able to govern translation and rotation simultaneously during a docking phase or any other attitude and position control, $n \geq 7$ thrusters are required. The appropriate control signals must then meet the requirements

- of being positive
- of being smaller than a maximum value (in general equal 1).

Furthermore, with more than 7 thrusters additionally an effectiveness criterion, which corresponds in general to fuel consumption, can be optimized.

The mathematical formulation thus leads to the following linear optimization problem (LOP):

Searching: thruster control a, for which the following applies:

(a) $0 \leq a \leq f$

(b) $T_c a = r$ dim $T_c = 6 \times n$,

(1) so that the following effectiveness criterion is met:

c) $\Sigma g_i a_i = g^T a \rightarrow \min$

wherein

a: control vector

f: vector that contains the maximum values of a_i

g: vector of the weight factors

$T_c = \left(\frac{f_1, \ldots, f_n}{t_{c1}, \ldots, t_{cn}} \right)$: weight and momental matrix

r: vector that contains the required forces and moments.

To apply all LOP solution methods, initially one permissible solution must be found in an initialization phase, i.e. a vector a_z, which fulfills (1a) and (1b). With the so-called singular value decomposition (abbreviated SVD) of T_c

$T_c = V \Sigma U_1^T$

with

V:
U = (U_1, U_2): } orthogonal matrix

$\rightarrow T_c U_2 = 0$

Σ = diag $\sigma l > 0$: diagonal matrix of the singular values

all solutions of (1b) can be described as

$a_0 = U_1 \Sigma^{-1} V^T r + U_2 c$

(2) =: U_1 s + v

(2) reveals the following:

(i) to realize random forces and moments all (σl) must be greater than zero, i.e. T_c must have full rank

(ii) the first addend is completely determined by r and represents the solution with minimal norm of (1b)

(iii) the second addend with the vector c that still has to be determined serves to fulfill the boundary condition (1a) and to minimize (1c)

From the fact that with the thruster set it must be possible to realize both positive and negative r, it results from (2) that c_1 and c_2 must exist so that

(a) $a_1 := U_1 s_1 + U_2 c_1 \geq 0$

(b) $a_2 := U_1 (-s_1) + U_2 c_2 \geq 0$

(3)

(c) $\rightarrow U_2 (c_1 + c_2) =: U_2 c_p > 0$

i.e. the existence of a random number of vectors c_p with

(4) $v_p = U_2 c_p > 0$

is guaranteed. After selecting a certain (a priori fixed) v_p, a_0 can be made positive pursuant to

(a) $a_0 = U_1 s + v_p k_1 \geq 0$,

(5)

(b) $k_1 = \max_i \frac{(-U_1 s)i + \varepsilon}{vp_i}$, $\varepsilon \approx 0.04 > 0$

wherein ε for numerical reasons was introduced for application of the following optimization steps.

For large right sides r it is possible that (1a, b) has no solution $a \leq f$, therefore the problem that is expanded by x_s is considered

(a) $T_c a = r\,(1 - x_s)$, (b) $0 \leq a \leq f$

$0 \leq x_s \leq 1$

(6) (c) $\rightarrow (T_c, r) \begin{pmatrix} a \\ x_s \end{pmatrix} = r$

(d) $g^T a + g_s x_s \rightarrow \min$

(e) $g_s \geq g^T f$, so that x_s becomes zero if r is not too large

This now also allows the upper bound to be adhered to and allows the required permissible starting value for a to be calculated for

(a) $a_z = a_0\,(1 - x_{sz}) = U_1\, s\,(1 - x_{sz}) + v_p\, k_1\,(1 - x_{sz})$

(7) (b) $x_{sz} = \begin{cases} w, & w > 0 \\ 0, & w \leq 0 \end{cases}$

(c) $w = \max_i \left\{ 1 - \dfrac{f_i - \varepsilon}{a_{0i}} \right\}$

ε here represents a first, for example infinitesimal, distance.

All subsequent considerations relate to the expanded system (6), wherein however the original description pursuant to (1) is maintained for reasons of simplicity.

To resolve the LOP a second procedural step now follows, namely an optimization of the efficiency criterion (1c) and/or (6d), which is performed iteratively as follows:

(a) $a_{i+1} = a_i - D_i U_2^{(i)} U_2^{(i)T} D_I g \bullet k$

$= a_I - v_{gI} \bullet k$

In this v_{gi} represents the iteration gradient, which is scaled in every iteration step, i.e. in each iteration step the gradient direction is newly determined. Moreover, k represents a limiting factor for the iteration step width, which is determined as follows:

(b) $k = \min (k_u, k_0) (1 - \varepsilon)$

(8)

(c) $$k_o = \min_{v_{gi}^{(j)} < 0} \left(\frac{a_i^{(j)} - ff}{v_{gi}^{(j)}} \right) ; \quad k_u = \min_{v_{gi}^{(j)} > 0} \left(\frac{a_i^{(j)}}{v_{gi}^{(j)}} \right)$$

This selection of k while taking the boundary value condition $0 \leq a \leq f$ into account ensures that a_{i+1} remains permissible.

(d) $$D_i = diag \left[a_i^{(j)} \left(1 - \frac{a_i^{(j)}}{f_j} \right) \right], j = 1,.., n+1$$

(e) $U_2^{(I)}$: zero space vectors of $T_c D_i = V^{(I)} \Sigma^{(i)} U_1^{(I)T}$

$a_1 = a_z$, calculated pursuant to (7).

The essential idea in (8) is the constant scaling of the problem with D_I and the subsequent continuation into the thus modified, on $U_2^{(i)}$ projected, negative gradient direction, wherein, due to the familiar structure of the problem as a convex linear optimization problem with an optimal solution on the boundary, it is guaranteed that the efficiency criterion is reduced in every iteration step. The iteration is preferably interrupted when the amount of $v_{gI} \cdot k$ drops below a specified threshold as a second distance, i.e. a_I hardly changes any more.

A particular expansion of the present method as compared with the method at Karmakar consists in taking an additional boundary value problem into account with every iteration step, here the inclusion of the upper bound f (upper bound problem) by adding the second factor in D_i and taking the upper bound f into consideration within the framework of the term k_0 in the calculation of k pursuant to (8b). Up to now, in the Karmakar methods usually complex expansions of the linear optimization problem with slack variables were offered with the disadvantage that then the dimension of the problem that needs to be solved is increased considerably. Here the present method represents an essential simplification. Additionally it eliminates the very complex determination of a permissible solution in the initialization phase, as practiced in Karmakar, through the suggested initialization phase, which is better adapted to the present problem.

Another advantageous procedural step of the method described here is thus in the constant calculation of v_{gi}, which preferably occurs not through the SVD of T_cD_i, but due to (here we use the following simplified depiction: T_c for T_cD_i, D for D_i)

(a) $M = T_cT_c^T = V \Sigma^2 V^T$

$\rightarrow$

(b) $U_1U_1^T = T_c^T M^{-1} T_c$

$v_g = D\, U_2U_2^T\, D_g = D(I - U_1U_1^T)\, D_g$

$\vee$ b

(9) $= D\,(I - T_c^T M^{-1} T_c)\, b$

(c) $= D\,(b - T_c^T x)$

with

(d) $Mx = T_cb$

via the solution from (9d) for x by means of a Gauss elimination. This method is clearly faster than an SVD. For a_o as well, U_1 is not determined, but U_{1s} is calculated directly pursuant to

(e) $U_{1s} = U_1 \Sigma^{-1} V^T r = T_c^T M^{-1} r$

Thus the method consists in the execution of the calculation steps

- equation (5) with (9e)
- equation (7) } initialization
- equation (8) with (9c, d) } iteration

Finally additional advantages of the suggested method compared with a simplex method pursuant to the state of the art are summarized in the following:

- a calculation of a permissible solution can occur with minimal effort (even a_z could already be used as a thruster selection, although it would not be fuel-optimized),

- the subsequent optimization can be limited to a few steps if necessary when e.g. the power of the on-board computer is limited,

- experience shows that the optimum on average is achieved with considerably fewer arithmetic operations and generally requires less memory than with a simplex method,

- in the case of a thruster failure, T_c must be reduced only by the appropriate column, and an appropriate vector v_p must be stored, i.e. in this case only minimal additional memory is required, in contrast to the state of the art,

- by including the upper bound problem in practice problems are avoided especially with limited thruster adjusting capacity.